Sliding Mode Control Using Modified Rodrigues Parameters

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Introduction

SPACECRAFT pointing poses a complex problem involving nonlinear dynamics with either linear and/or nonlinear control laws. Primary control actuators usually include thrusters for rapid and coarse attitude maneuvers and reaction wheels for slow and precise attitude maneuvers. Other types of control mechanisms include gravity-gradient stabilization and magnetic torquer assemblies. Control algorithms can be divided into open-loop systems and closed-loop (feedback) systems. Open-loop systems usually require a predetermined pointing maneuver and are typically determined using optimal control techniques that involve the solution of a two-point boundary value problem. An example of open-loop control is the time-optimal attitude maneuver (e.g., see the excellent survey paper by Scrivener and Thompson[1]). Closed-loop systems can provide robustness with respect to spacecraft modeling uncertainties and unexpected disturbances.

The control technique used in this Note is based on sliding mode (variable structure) control.[2] This type of control has been successfully applied for spacecraft pointing and regulation using both a Rodrigues (Gibbs vector) representation[3] and a quaternion representation.[4] An advantage of the quaternion representation is that singularities in the kinematic equations can be avoided. However, the use of quaternions requires an extra parameter, which leads to a nonminimal parameterization. The Rodrigues parameters provide a minimal (i.e., three-dimensional) parameterization. However, a singularity exists for 180-deg rotations, which hinders this parameterization for extremely large angle rotations. This difficulty may be overcome by applying successive rotations, each less than 180 deg. However, the overall maneuver may require extra control authority and power requirements, which may not be necessary. In this Note, a sliding mode controller is developed based on the modified Rodrigues parameters.[5,6] Advantages of using this attitude representation include the following: 1) rotations of up to 360 deg are possible, and 2) the parameters form a minimal parameterization.

Attitude Kinematics and Dynamics

In this section, a brief review of the kinematic equations of motion using the modified Rodrigues parameters is shown. This parameterization is derived by applying a stereographic projection of the quaternions. The quaternion representation is given by

$$q \equiv \begin{bmatrix} q_{13} \\ q_4 \end{bmatrix} \tag{1}$$

with

$$q_{13} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \hat{n} \sin\left(\frac{\theta}{2}\right) \tag{2a}$$

$$q_4 = \cos(\theta/2) \tag{2b}$$

where $\hat{n}$ is a unit vector corresponding to the axis of rotation and θ is the angle of rotation. The modified Rodrigues parameters are defined by[5]

$$p = q_{13}/(1 + q_4) = \hat{n}\tan(\theta/4) \tag{3}$$

where p is a 3×1 vector. The kinematic equations of motion are derived by using the spacecraft's angular velocity (ω), given by[6]

$$\dot{p} = \tfrac{1}{4}\left\{(1 - p^T p) I_{3\times3} + 2[p\times] + 2pp^T\right\}\omega \tag{4}$$

where $I_{3\times3}$ is a 3×3 identity matrix and $[p\times]$ is a 3×3 cross-product matrix defined by

$$[p\times] \equiv \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \tag{5}$$

The dynamic equation of motion is given by Euler's equation, defined by

$$\dot{\omega} = J^{-1}[J\omega\times]\omega + J^{-1}u \tag{6}$$

where J is the spacecraft's inertia (3×3) matrix and u is a torque input.

Sliding Mode Controller

In this section a sliding mode controller is developed using the modified Rodrigues parameters. It is assumed that measurements of both the spacecraft attitude and angular rate are available, which may be provided by a Kalman filter. The nonlinear model for spacecraft motion is summarized by

$$\dot{p} = F(p)\omega \tag{7a}$$

$$\dot{\omega} = f(\omega) + J^{-1}u \tag{7b}$$

where

$$F(p) \equiv \tfrac{1}{4}\left\{(1 - p^T p) I_{3\times3} + 2[p\times] + 2pp^T\right\} \tag{8a}$$

$$f(\omega) \equiv J^{-1}[J\omega\times]\omega \tag{8b}$$

Under ideal conditions, the state trajectories move onto a sliding manifold ($s = 0$), where s is given by

$$s \equiv \omega - m(p) \tag{9}$$

The quantity $m(p)$ is obtained using a desired vector field from the kinematic relations,[3] given by

$$m(p) = F^{-1}(p)d(p) \tag{10}$$

where

$$F^{-1}(p) = 4(1 + p^T p)^{-2}\left\{(1 - p^T p) I_{3\times3} - 2[p\times] + 2pp^T\right\} \tag{11}$$

The quantity $d(p)$ is formed by allowing a linear behavior in the sliding motion, given by

$$d(p) = \Lambda(p - p_d) \tag{12}$$

where p_d is the desired reference trajectory and Λ is a diagonal matrix with negative elements. This allows for decoupled sliding motions and exponential convergence toward the final desired orientation. The sliding mode controller that produces a negative definite derivative of the Lyapunov function $s^T s$ is given by[3]

$$u = -J\left\{f(\omega) - \frac{\partial m}{\partial p}[F(p)m(p) + F(p)s] + K\,\mathrm{sat}(s, \varepsilon)\right\} \tag{13}$$

where K is a 3×3 positive definite, diagonal matrix. The saturation function is used to minimize chattering in the control torques. This function is defined by

$$\mathrm{sat}(s_i, \varepsilon) \equiv \begin{cases} 1 & \text{for } s_i > \varepsilon \\ s_i/\varepsilon & \text{for } |s_i| \le \varepsilon \\ -1 & \text{for } s_i < -\varepsilon \end{cases} \qquad i = 1, 2, 3 \tag{14}$$

where ε is a small positive quantity.

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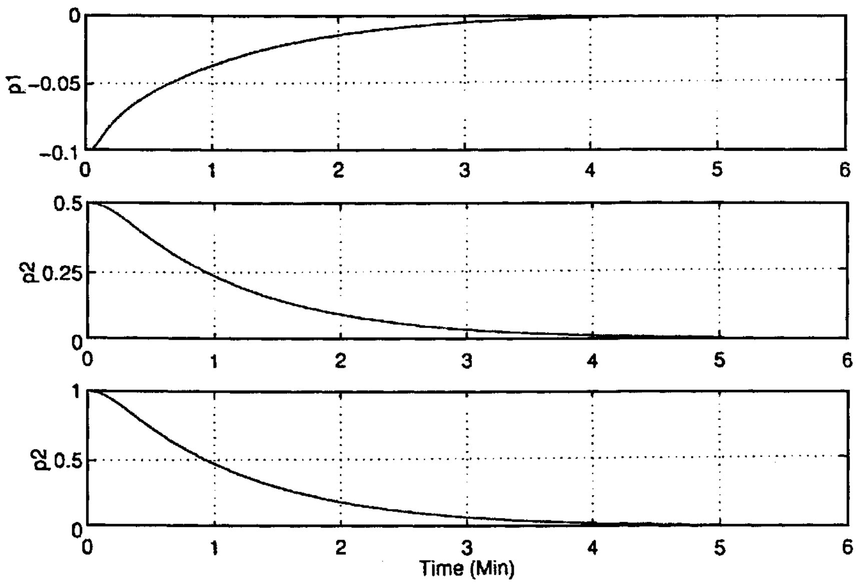

Fig. 1 Plot of closed-loop modified Rodrigues trajectories.

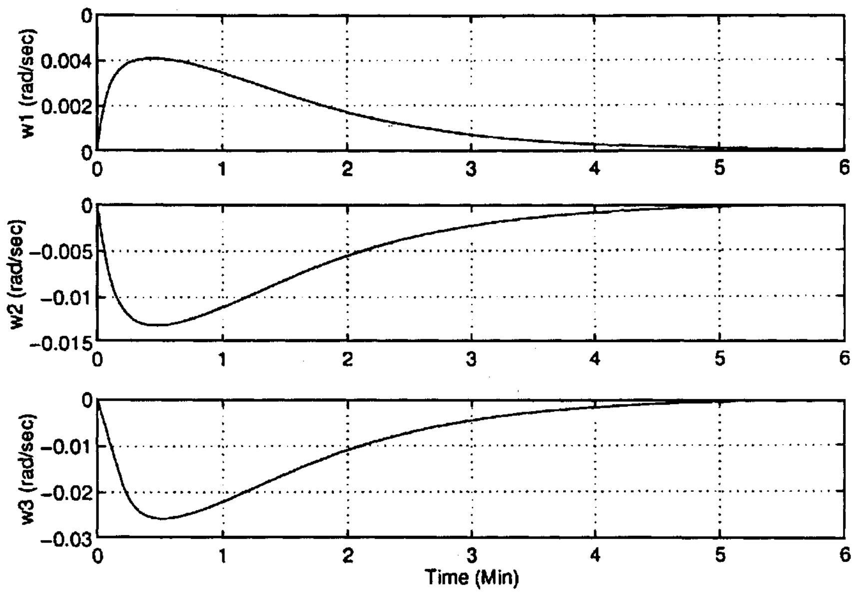

Fig. 2 Plot of closed-loop angular velocity trajectories.

Regulation

The regulation problem requires that the final spacecraft position be zero (i.e., unity quaternion). This corresponds to desired modified Rodrigues parameters given by $p_d = 0$. Also, the Λ matrix in Eq. (12) is assumed to be given by a scalar (λ) times the identity matrix, which leads to

$$m(p) = 4\lambda(1 + p^T p)^{-2}\left\{(1 - p^T p)I_{3\times 3} - 2[p\times] + 2pp^T\right\}p \quad (15)$$

This relation can be simplified significantly by applying some cross-product relations, which leads to

$$m(p) = 4\lambda(1 + p^T p)^{-1} p \quad (16)$$

The partial derivative of Eq. (16) with respect to p is given by

$$\frac{\partial m}{\partial p} = 4\lambda(1 + p^T p)^{-1}\left\{I_{3\times 3} - 2(1 + p^T p)^{-1} pp^T\right\} \quad (17)$$

Tracking

The tracking problem requires the system attitude to follow a desired reference trajectory. Equation (10) for the tracking problem is derived to be

$$m(p) = 4\lambda(1 + p^T p)^{-1} p - 4\lambda(1 + p^T p)^{-2} \times \left\{(1 - p^T p)I_{3\times 3} - 2[p\times] + 2pp^T\right\}p_d \quad (18)$$

The partial derivative of Eq. (18) with respect to p is given by

$$\frac{\partial m}{\partial p} = 4\lambda(1 + p^T p)^{-1}\left\{I_{3\times 3} - 2(1 + p^T p)^{-1} pp^T\right\} - 8\lambda(1 + p^T p)^{-2}\left\{pp_d^T - p_d p^T + [p_d\times] + (p_d^T p)I_{3\times 3}\right\} + 16\lambda(1 + p^T p)^{-3}\left\{(1 - p^T p)I_{3\times 3} - 2[p\times] + 2pp^T\right\}p_d p^T \quad (19)$$

Spacecraft Simulation

The spacecraft simulation involves a multi-axis rest-to-rest maneuver. The inertia matrix for the simulated spacecraft is given by[3]

$$J = \mathrm{diag}[114, \quad 86, \quad 87]\,\mathrm{kg\text{-}m^2} \quad (20)$$

The initial conditions for the angular velocity are set to zero, and the initial conditions for the modified Rodrigues parameters are given by

$$p(t_0) = [-0.1 \quad 0.5 \quad 1.0]^T \quad (21)$$

The desired attitude parameters are set to zero (i.e., the regulation case). The diagonal elements of K in Eq. (13) are all set to 0.0015,

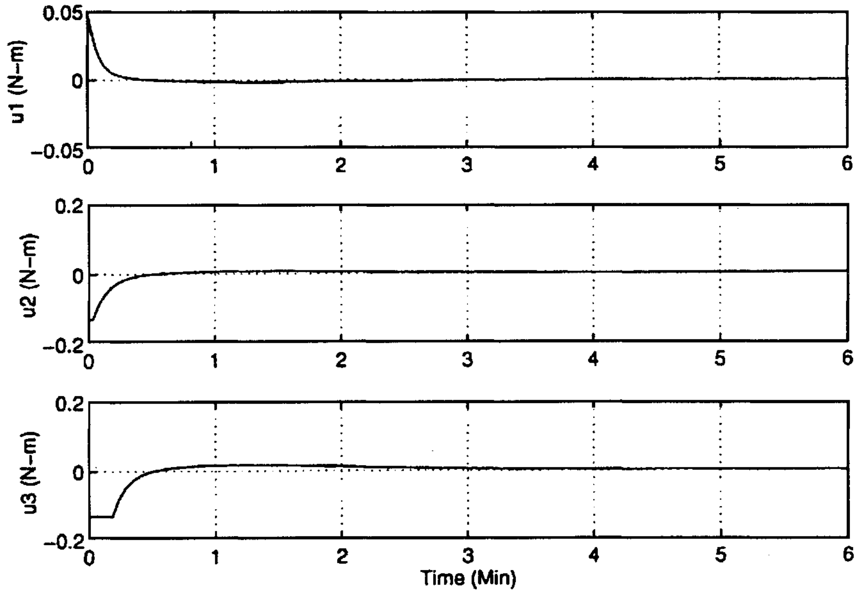

Fig. 3 Plot of applied control torques.

and the constant λ is set to $-0.015\ s^{-1}$. The parameter ε in the saturation controller is set to 0.01. Also, the control torques are limited to 1.0 N-m. A plot of the closed-loop modified Rodrigues parameters is shown in Fig. 1. Also, plots of the angular velocity trajectories and applied control torques are shown in Figs. 2 and 3, respectively. Using Eq. (3), the rotation for the initial conditions in Eq. (21) is approximately 193 deg. Therefore, converting the modified Rodrigues parameters shown in Fig. 1 to quaternions[6] reveals that the scalar (fourth) quaternion crosses the zero point. Therefore, the Gibbs vector control formulation in Ref. 3 becomes singular but is easily handled by the modified Rodrigues parameter control formulation.

Conclusions

In this Note, a sliding mode controller was developed for attitude pointing using the modified Rodrigues parameters. The modified Rodriques parameters represent a minimal parameterization with a singularity at 360 deg. These parameters avoid the normalization constraint associated with the quaternion parameterization and allow for rotations of greater than 180 deg for which the Gibbs vector parameterization becomes singular. Simulation results indicate that the new algorithm was able to accurately control the attitude of a spacecraft for large-angle maneuvers.

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References

[1]Scrivener, S. L., and Thompson, R. C., "Survey of Time-Optimal Attitude Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 2, 1994, pp. 225–233.

[2]Utkin, V. I., "Variable Structure Systems with Sliding Modes," *IEEE Transactions on Automatic Control*, Vol. AC-22, No. 2, 1977, pp. 212–222.

[3]Ramirez, H. S., and Dwyer, T. A. W., "Variable Structure Control of Spacecraft Reorientation Maneuvers," *Proceedings of the AIAA Guidance, Navigation, and Control Conference* (Williamsburg, VA), AIAA, New York, 1986, pp. 88–96 (AIAA Paper 86-1987).

[4]Vadali, S. R., "Variable Structure Control of Spacecraft Large Angle Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, 1986, pp. 235–239.

[5]Marandi, S. R., and Modi, V. J., "A Preferred Coordinate System and Associated Orientation Representation in Attitude Dynamics," *Acta Astronautica*, Vol. 15, No. 11, 1987, pp. 833–843.

[6]Shuster, M. D., "A Survey of Attitude Representations," *Journal of the Astronautical Sciences*, Vol. 41, No. 4, 1993, pp. 439–517.

Application of a Learning Control Algorithm to a Rotor Blade Tab

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Introduction

THE suppression of vibration is of continuing interest in rotorcraft technology. Because of periodic excitation inherent in forward flight, a rotorcraft is subject to varying dynamic and aerodynamic loads. These variable loads act on the main rotor, and attempts to alleviate them provide the motivation for many studies of various rotor design concepts and of the application of active control either to rotor pitch (higher harmonic control and individual blade control concepts)[1] or to specially designed additional devices.[2]

A tab mounted at the blade trailing edge has generated much interest recently for additional rotor control. This has arisen from prospects of providing the driving mechanism for the tabs through smart structure technology.[3] Several analytical and experimental studies have been carried out to obtain insight into different aspects of its application.[4] Tabs driven by piezoelectric benders were tested experimentally on a rotor model in hover.[5]

Physical phenomena involved in applications of the "smart tab" are aeroelastic [i.e., including both dynamic (inertia and elastic loads)] and aerodynamic phenomena.

To achieve the required goal a proper control strategy should be applied to the system. Consideration has been given to open loop in Ref. 6, and control algorithms of mainly optimal linear quadratic

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